# 1. Basic Nuclear Processes in Radioactive Sources

Radioactive sources provide a convenient means of testing and calibrating detectors and are essential tools in both the nuclear and high energy physics laboratory. An understanding of the basic nuclear processes in radioactive sources is a necessity, therefore, before beginning to work in the laboratory. We shall begin this book then by briefly reviewing these processes and describing the characteristics of the resulting radiations. A more detailed discussion, of course, is better suited for a nuclear theory course and we refer the reader to any of the standard nuclear physics texts for more information.

Nuclei can undergo a variety of processes resulting in the emission of radiation of some form. We can divide the processes into two categories: radioactivity and nuclear reactions. In a radioactive transformation, the nucleus spontaneously disintegrates to a different species of nuclei or to a lower energy state of the same nucleus with the emission of radiation of some sort. The majority of radiation sources found in the laboratory are of this type. In a nuclear reaction, the nucleus interacts with another particle or nucleus with the subsequent emission of radiation as one of its final products. In many cases, as well, some of the products are nuclei which further undergo radioactive disintegration.

The radiation emitted in both of these processes may be electromagnetic or corpuscular. The electromagnetic radiations consist of x-rays and  $\gamma$ -rays while the corpuscular emissions include  $\alpha$ -particles,  $\beta$ -electrons and positrons, internal conversion electrons, Auger electrons, neutrons, protons, and fission fragments, among others. While most of these radiations originate in the nucleus itself, some also arise from the electron cloud surrounding the nucleus. Indeed, the nucleus should not be considered as a system isolated from the rest of the atom. Excitations which arise therein may, in fact, make themselves directly felt in the electron cloud, as in the case of internal conversion, and/or indirectly as in the emission of characteristic x-rays following electron capture. Except for their energy characteristics, these radiations are indistinguishable from those arising in the nucleus.

Table 1.1 summarizes some of the more common types of radiation found in laboratory sources. Each radiation type is characterized by an energy spectrum which is indicative of the nuclear process underlying it. Note also that a radioactive source may emit several different types of radiation at the same time. This can arise from the fact that the nuclear isotope in question undergoes several different modes of decay. For example, a  $^{137}$ Cs nucleus can de-excite through either  $\gamma$ -ray emission or internal conversion. The output of a given  $^{137}$ Cs sample, therefore, will consist of both photons and electrons in a proportion equal to the relative probabilities for the two decay modes. A more common occurrence, however, is that the daughter nucleus is also radioactive, so that its radiation is also added to the emitted output. This is the case with many  $\beta$ -sources, where the  $\beta$ -disintegration results in an excited daughter nucleus which then immediately decays by  $\gamma$ -emission.

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Table 1.1. Characteristics of nuclear radiations

Type	Origin	Process	Charge	Mass [MeV]	Spectrum (energy)
α-particles	Nucleus	Nuclear decay or reaction	+2	3727.33	Discrete [MeV]
β <sup>-</sup> -rays	Nucleus	Nuclear decay	-1	0.511	Continuous [keV – MeV)
β <sup>+</sup> -rays (positrons)	Nuclear	Nuclear decay	+1	0.511	Continuous [keV – MeV]
γ-rays	Nucleus	Nuclear deexcitation	0	0	Discrete [keV – MeV]
x-rays	Electron cloud	Atomic deexcitation	0	0	Discrete [eV – keV]
Internal conversion electrons	Electron cloud	Nuclear deexcitation	-1	0.511	Discrete [high keV]
Auger	Electron cloud	Atomic deexcitation	-1	0.511	Discrete [eV – keV]
Neutrons	Nucleus	Nuclear reaction	0	939.57	Continuous or discrete [keV – MeV]
Fission fragments	Nucleus	Fission	=20	80 – 160	Continuous 30 – 150 MeV

## 1.1 Nuclear Level Diagrams

Throughout this text, we will be making use of nuclear energy level diagrams, which provide a compact and convenient way of representing the changes which occur in nuclear transformations. These are usually plotted in the following way. For a given nucleus with atomic number Z and mass A, the energy levels are plotted as horizontal lines on some arbitrary vertical scale. The spin and parity of each of these states may also be indicated. Keeping the same mass number A, the energy levels of neighboring nuclei (Z-1,A), (Z+1,A), ... are now also plotted on this energy scale with Z ordered in the horizontal direction, as illustrated in Fig. 1.1. This reflects the fact that nuclei with different Z but the same A may simply be treated as different states of a system of A nucleons. The relation of the energy levels of a nucleus (Z,A) to other nuclei in the same A system is therefore made apparent. With the exception of  $\alpha$ -decay, radioactive decay may now be viewed as simply a transition from a higher energy state to a lower energy state within the same system of A nucleons. For example, consider the  $\beta$ -decay process, which will be discussed in the next section. This reaction involves the decay

$$(Z, A) \rightarrow (Z+1, A) + e^{-} + \bar{v}$$
,

where the final state in the nucleus (Z+1, A) may be the ground state or some excited state. This is shown in Fig. 1.1 by the arrow descending to the right. The atomic number Z increases by one, but A remains constant. The changes that occur can be immedi-

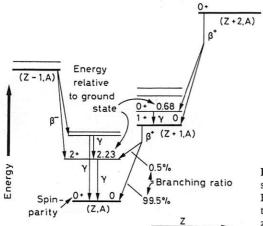


Fig. 1.1. Nuclear level diagrams conveniently represent the transitions which can occur between nuclei. For a system of A nucleons, energy is represented on the vertical scale while atomic number is on the horizontal scale

ately seen: for example, the energy available for the reaction as given by the difference in height between the two levels, the spin and parity changes, etc. If it is possible for the same initial state to make transitions to several different final states, this can also be represented by several arrows emanating from the initial state to the various possible final states. The relative probability of each decay branch (i.e., the branching ratio) may also be indicated next to the corresponding arrow.

In a similar way, transitions which follow the first may also be diagrammed. For example, suppose the final state of the above example is an excited state of the (Z+1,A) nucleus, it may then make a gamma transition to the ground state or to another excited state. This type of transition is indicated by a vertical line since Z remains unchanged (see Fig. 1.1). Other transitions, such as a further  $\beta$ -decay or some other process, may be represented in a similar manner. In this way, the types of radiation emitted by a particular radioactive source and their origins may be easily displayed.

Several tabulations of the radioactive isotopes and their level diagrams as deduced from experiment are available for reference purposes. The most complete of these is the *Table of Isotopes* edited by *Lederer* and *Shirley* [1.1] which is updated periodically. We urge the reader to become familiar with their interpretation.

## 1.2 Alpha Decay

Alpha particles are <sup>4</sup>He nuclei, i.e., a bound system of two protons and two neutrons, and are generally emitted by very heavy nuclei containing too many nucleons to remain stable. The emission of such a nucleon cluster as a whole rather than the emission of single nucleons is energetically more advantageous because of the particularly high binding energy of the  $\alpha$ -particle. The parent nucleus (Z, A) in the reaction is thus transformed via

$$(Z, A) \to (Z-2, A-4) + \alpha$$
. (1.1)

Theoretically, the process was first explained by Gamow and Condon and by Gurney as the tunneling of the  $\alpha$ -particle through the potential barrier of the nucleus. Alpha

particles, therefore, show a monoenergetic energy spectrum. As well, since barrier transmission is dependent on energy, all  $\alpha$ -sources are generally limited to the range  $\approx 4-6$  MeV with the higher energy sources having the higher transmission probability and thus the shorter half-life. For this reason also, most  $\alpha$ -decays are directly to the ground state of the daughter nucleus since this involves the highest energy change. Decays to excited states of the daughter nucleus are nevertheless possible, and in such nuclei, the energy spectrum shows several monoenergetic lines each corresponding to a decay to one of these states. Some of the more commonly used sources are listed below in Table 1.2.

Table 1.2. Characteristics of some alpha emitters

	Energies [MeV]	Branching
433 yrs.	5.486 5.443	85% 12.8%
138 days	5.305	100%
163 days	6.113 6.070	74% 26%
	138 days	5.443 138 days 5.305 163 days 6.113

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Because of its double charge, +2e, alpha particles have a very high rate of energy loss in matter. The range of a 5 MeV  $\alpha$ -particle in air is only a few centimeters, for example. For this reason it is necessary to make  $\alpha$  sources as thin as possible in order to minimize energy loss and particle absorption. Most  $\alpha$ -sources are made, in fact, by depositing the isotope on the surface of a suitable backing material and protecting it with an extremely thin layer of metal foil.

## 1.3 Beta Decay

Beta particles are fast electrons or positrons which result from the weak-interaction decay of a neutron or proton in nuclei which contain an excess of the respective nucleon. In a neutron-rich nucleus, for example, a neutron can transform itself into a proton via the process

$$n \to p + e^- + \bar{\nu}, \qquad (1.2)$$

where an electron and antineutrino are emitted. (The proton remains bound to the nucleus.) The daughter nucleus now contains one extra proton so that its atomic number is increased by 1.

Similarly, in nuclei with too many protons, the decay

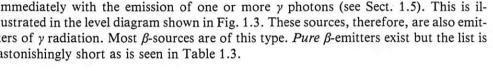
$$p \to n + e^+ + \nu \tag{1.3}$$

can occur, where a positron and a neutrino are now emitted and the atomic number is decreased by 1. Both are mediated by the same weak interaction.

A basic characteristic of the  $\beta$ -decay process is the continuous energy spectrum of the  $\beta$ -particle. This is because the available energy for the decay (the Q-value) is shared

between the \(\beta\)-particle and the neutrino (or antineutrino) which usually goes undetected. A typical spectrum is shown in Fig. 1.2. If the small recoil energy of the daughter nucleus is ignored, the maximum energy of this spectrum should correspond to the Qvalue for the reaction. For most beta sources, this maximum value ranges from a few tens of keV to a few MeV.

In very many  $\beta$ -sources, the daughter nucleus is left in an excited state which decays immediately with the emission of one or more y photons (see Sect. 1.5). This is illustrated in the level diagram shown in Fig. 1.3. These sources, therefore, are also emitters of  $\gamma$  radiation. Most  $\beta$ -sources are of this type. Pure  $\beta$ -emitters exist but the list is astonishingly short as is seen in Table 1.3.



**Table 1.3.** List of pure  $\beta^-$  emitters

Source	Half-life	$E_{\rm max}$ [MeV]
<sup>3</sup> H	12.26 yr	0.0186
<sup>14</sup> C	5730 yr	0.156
$^{32}P$	14.28 d	1.710
<sup>33</sup> P	24.4 d	0.248
35S	87.9 d	0.167
36Cl	$3.08 \times 10^5 \text{ yr}$	0.714
<sup>45</sup> Ca	165 d	0.252
63Ni	92 yr	0.067
90Sr/90Y	27.7 yr/64 h	0.546/2.27
<sup>99</sup> Tc	$2.12 \times 10^{5}  \text{yr}$	0.292
147Pm	2.62 yr	0.224
<sup>204</sup> Tl	3.81 yr	0.766

Some  $\beta$  sources may also have more than one decay branch, i.e., they can decay to different excited states of the daughter nucleus. Each branch constitutes a separate  $\beta$ decay with an end-point energy corresponding to the energy difference between the initial and final states and is in competition with the other branches. The total  $\beta$ -spectrum from such a source is then a superposition of all the branches weighted by their respective decay probabilities.

Since electrons lose their energy relatively easily in matter, it is important that  $\beta$ sources be thin in order to allow the  $\beta$ 's to escape with a minimum of energy loss and absorption. This is particularly important for positron sources since the positron can annihilate with the electrons in the source material or surrounding container. A too

$$\frac{5+5.2 \text{ yr}}{60 \text{ Co}_{4}} + \frac{\beta}{1.33} = \frac{2.505}{2.505}$$

$$\frac{7/2+27 \text{ yr}}{137 \text{ Cs}} + \frac{92\%}{0.662}$$

$$\frac{7/2-270d}{137_{Ba}}$$

$$\frac{5/2-EC}{0.136} + \frac{5}{10.0144}$$

$$\frac{5/2-EC}{10.00144} + \frac{5}{10.00144}$$

$$\frac{2+5}{57} + \frac{2.6 \text{ yr}}{1.277}$$

$$\frac{2+5}{1.277} + \frac{2.6 \text{ yr}}{1.277}$$
Fig. 1.3. Nuclear level diagrams of a few common gamma sources

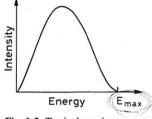


Fig. 1.2. Typical continuous energy spectrum of beta decay electrons

thick  $\beta^+$  source will exhibit a distorted  $\beta^+$  spectrum and an enormous background of 511 keV annihilation photons.

## 1.4 Electron Capture (EC)

As an alternative to  $\beta^+$  emission, proton-rich nuclei may also transform themselves via the capture of an electron from one of the atomic orbitals

$$e^- + p \rightarrow n + \nu \,. \tag{1.4}$$

This reaction is essentially the same as  $\beta^+$ -decay but with the  $\beta$  particle transposed to the left side. The nuclear level diagram for EC is therefore identical to that for  $\beta^+$  emission. Since only the neutrino is emitted, electron capture would seem to be a reaction almost impossible to observe, given the well-known difficulty of detecting such a particle! The capture of the electron, however, leaves a hole in the atomic shell which is filled by another atomic electron, giving rise to the emission of a characteristic x-ray or Auger electrons (see Sect. 1.8). These radiations are, of course, much more amenable to detection and can be used to signal the capture reaction. In general, it is the K electron which is most likely captured, although, L-capture is also possible but with a much smaller probability.

#### 1.5 Gamma Emission

Like the electron shell structure of the atom, the nucleus is also characterized by discrete energy levels. Transitions between these levels can be made by the emission (or absorption) of electromagnetic radiation of the correct energy, i.e., with an energy equal to the energy difference between the levels participating in the transition. The energies of these photons, from a few hundred keV to a few MeV, characterize the high binding energy of nuclei. These high-energy photons were historically named  $\gamma$ -rays, and, like atoms, show spectral lines characteristic of the emitting nucleus. Level diagrams illustrating the specific energy structure of some typical  $\gamma$ -ray sources are shown in Fig. 1.3.

Most  $\gamma$ -sources are "placed" in their excited states as the result of a  $\beta$ -disintegration, although excited nuclear states are often created in nuclear reactions also. Since electrons and positrons are more easily absorbed in matter, the  $\beta$ -particles in such sources can be "filtered" out by enveloping them with sufficient absorbing material, leaving only the more penetrating  $\gamma$ -ray.

#### 1.5.1 Isomeric States

Although most excited states in nuclei make almost immediate transitions to a lower state, some nuclear states may live very much longer. Their de-excitation is usually hindered by a large spin difference between levels (i.e., a *forbidden* transition) resulting in lifetimes ranging from seconds to years. A nuclide which is "trapped" in one of these metastable states will thus show radioactive properties different from those in more normal states. Such nuclei are called *isomers* and are denoted by an m next to the mass number in their formulae, e.g.  $^{60m}$ Co or  $^{69m}$ Zn.

## 1.6 Annihilation Radiation

Another source of high-energy photons is the annihilation of positrons. If a positron source such as  $^{22}$ Na is enclosed or allowed to irradiate an absorbing material, the positrons will annihilate with the absorber electrons to produce two photons, each with an energy equal to the electron mass: 511 keV. In order to conserve momentum, these two photons are always emitted in opposite directions. The  $\gamma$  spectrum from a thick positron source will thus show a peak at 511 keV (corresponding to the detection of one of the annihilation photons) in addition to the peaks characteristic of transitions in the daughter nucleus. Figure 1.4 shows the spectrum observed with a thick  $^{22}$ Na source.

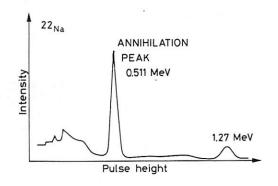


Fig. 1.4. Gamma-ray spectrum of a <sup>22</sup>Na source as observed with a NaI detector. Because of positron annihilation in the detector and the source itself, a peak at 511 keV is observed corresponding to the detection of one of the annihilation photons

#### 1.7 Internal Conversion

While the emission of a  $\gamma$ -ray is usually the most common mode of nuclear de-excitation, transitions may also occur through internal conversion. In this process, the nuclear excitation energy is directly transferred to an atomic electron rather than emitted as a photon. The electron is ejected with a kinetic energy equal to the excitation energy minus its atomic binding energy. Unlike  $\beta$ -decay, therefore, internal conversion electrons are monoenergetic having approximately the same energy as the competing  $\gamma$ 's, i.e., a few hundred keV to a few MeV.

While the K-shell electrons are the most likely electrons to be ejected, electrons in other orbitals may also receive the excitation energy. Thus, an internal conversion source will exhibit a group of internal conversion lines, their differences in energy being equal to the differences in the binding energies of their respective orbitals.

Internal conversion sources are one of the few nuclear sources of monoenergetic electrons and are thus very useful for calibration purposes. Some internal conversion sources readily found in the laboratory are given in Table 1.4.

Table 1.4. Some internal conversion sources

Source	Energies [keV]	
<sup>207</sup> Bi	480, 967, 1047	
<sup>137</sup> Cs	624	
<sup>113</sup> Sn	365	
<sup>133</sup> Ba	266, 319	

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# 1.8 Auger Electrons

As in internal conversion, an excitation which arises in the electron shell can also be transferred to an atomic electron rather than to a characteristic x-ray. Such a process can occur after a reaction such as electron-capture, for example. The electrons emitted are called *Auger* electrons and are monoenergetic. Like internal conversion lines they can occur in groups, however, their energies are more typical of atomic processes being not more than a few keV. They are thus very susceptible to self-absorption and are difficult to detect.

## 1.9 Neutron Sources

While it is possible to artificially produce isotopes which emit neutrons, natural neutron emitters which can be used practically in the lab do not exist. Laboratory neutron sources, instead, are based on either spontaneous fission or nuclear reactions.

## 1.9.1 Spontaneous Fission

Spontaneous fission can occur in many transuranium elements with the release of neutrons along with the fission fragments. These fragments, as well, can promptly decay emitting  $\beta$  and  $\gamma$  radiation. If the fission source is enveloped in a sufficiently thick container, however, much of this latter radiation can be absorbed leaving only the more penetrating neutrons.

The most common neutron source of this type is <sup>252</sup>Cf which has a half-life of 265 years. The energy spectrum of the neutrons is continuous up to about 10 MeV and exhibits a Maxwellian shape. Figure 1.5 shows this spectrum. The distribution is described very precisely by the form [1.2]

$$\frac{dN}{dE} = \sqrt{E} \exp\left(\frac{-E}{T}\right),\tag{1.5}$$

where T = 1.3 MeV for  $^{252}$ Cf.

#### 1.9.2 Nuclear Reactions

A more convenient method of producing neutrons is with the nuclear reactions  $(\alpha, n)$  or  $(\gamma, n)$ . Reactions of this type occur with many nuclei, however, only those with the highest yield are used. Such sources are generally made by mixing the target material with a suitably strong  $\alpha$  or  $\gamma$  emitter. The most common target material is beryllium. Under bombardment by  $\alpha$ 's, beryllium undergoes a number of reactions which lead to the production of free neutrons:

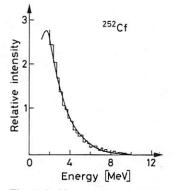


Fig. 1.5. Neutron energy spectrum from <sup>252</sup>Cf (from *Lorch* et al. [1.2]). The form of the spectrum can be described by a Maxwellian distribution

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<sup>&</sup>lt;sup>1</sup> A common method for denoting nuclear reactions is A(x, y)B where x is the bombarding particle, A the target nucleus, B the resulting nucleus and y the the outgoing particle or particles. Note that the ingoing and outgoing particles are always on the inside of the parentheses. The abbreviated notation (x, y), therefore, indicates any nuclear reaction in which x is the incident particle and y the resulting, outgoing particle.

$$\alpha + {}^{9}\text{Be} \rightarrow {}^{13}\text{C}^* \rightarrow \begin{cases} {}^{12}\text{C}^{(*)} + n \\ {}^{8}\text{Be} + \alpha + n \\ 3\alpha + n \end{cases}$$
 (1.6)

Here the excited compound nucleus  $^{13}C^*$  is formed which then decays through a variety of modes depending on the excitation energy. In general, the dominant reaction is the decay to  $^{12}C$  or to the 4.44 MeV excited state of  $^{12}C$ . With  $^{241}Am$  as an  $\alpha$ -source, a neutron yield of about 70 neutrons per  $10^6$   $\alpha$ 's [1.3] is generally obtained. With  $^{242}Cm$ , which emits  $\alpha$ 's at a higher energy, the yield is  $\approx 106$  neutrons/ $10^6$ - $\alpha$  [1.3]. Other ( $\alpha$ , n) neutron sources include  $^{238}Pu/Be$ ,  $^{226}Ra/Be$  and  $^{227}Ac/Be$ . Targets such as B, F, and Li are also used although the neutron yields are somewhat lower. The half-life of these sources, of course, depends on the half-life of the  $\alpha$ -emitter.

For incident  $\alpha$ 's of a fixed energy, the energy spectrum of neutrons emitted in these sources should theoretically show monoenergetic lines corresponding to the different transitions which are made. In mixed sources, however, there is a smearing of the alphaparticle spectrum due to energy loss, so that a large smearing in neutron energy results. There is also considerable Doppler broadening which can amount to as much as  $2 \, \text{MeV}$ . Figure 1.6 shows the energy spectrum of neutrons for several sources of this type.

In the case of the photo-reaction  $(\gamma, n)$ , only two target materials are suitable: beryllium and deuterium. The respective reactions are

$$^{9}\text{Be} + \gamma \rightarrow ^{8}\text{Be} + n$$
, (1.7)

$$^{2}H + \gamma \rightarrow ^{1}H + n$$
. (1.8)

These sources have the advantage of emitting neutrons which are more or less monoenergetic since the  $\gamma$ 's are not slowed down as in the case of  $\alpha$ 's. The neutrons are, of course, not strictly monoenergetic if one works out the kinematics; however, the spread is generally small. The disadvantage of these sources is that the reaction yield per  $\gamma$  is 1-2 orders of magnitude lower than that of the  $\alpha$ -type sources. As well, the nonreacting gammas are not absorbed as easily as  $\alpha$ -particles, so that these sources are also accompanied by a large background of  $\gamma$  radiation.

A more detailed description of these and other neutron sources may be found in the article by *Hanson* [1.4].

## 1.10 Source Activity Units

The *activity* or strength of a radioactive sample is defined as the mean number of *decay* processes it undergoes per unit time. This is an extrinsic quantity which depends on the amount of source material contained in the sample – the larger the sample the greater the total number of decays. Moreover, it should be noted that the activity of a source is not necessarily synonymous with the amount of radiation emitted per unit time by the source, although it is certainly related to it. For example, some nuclear transformations result in an unstable daughter nucleus which also disintegrates. Its radiations would then appear with the radiation from the original decay, but would not be included in the activity. Similarly, some nuclides decay through several competing processes, for example,  $\beta^+$ -emission or electron capture, where only a fraction of the decays appears

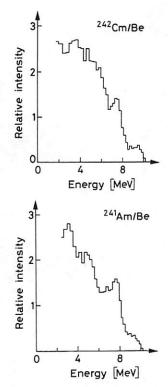


Fig. 1.6. Neutron energy spectrum from <sup>242</sup>Cm/Be and (*above*) and <sup>241</sup>Am/Be (*below*) sources (from *Lorch* et al. [1.2])

as a particular emitted radiation. The relation between radiation output and activity, in fact, depends on the specific nuclear decay scheme, and only in the case of a unique radiative transition is the activity identical to the radiation output.

Activity has traditionally been measured in units of *Curies* (Ci). Originally defined as the activity of 1 g of pure radium-226, this unit is equivalent to

1 Curie (Ci) = 
$$3.7 \times 10^{10}$$
 disintegrations/s (dps). (1.9)

This is, in fact, a very *large* unit and one generally works in the laboratory with sources on the order of tens or hundreds of microCuries ( $\mu$ Ci).

Because of its rather awkward definition in terms of dps, the Becauerel, defined as

1 Becquerel (Bq) = 1 disintegration/s 
$$(1.10)$$

is now recommended instead.

For the beginning student, it is important to distinguish units of *activity* from those of *dose* such as the Gray or the Sievert. These latter units essentially measure the effects of radiation *received* by an object or person, whereas the Curie or Becquerel are concerned with the disintegrations in the source itself. Units of dose will be treated in Chap. 3.

## 1.11 The Radioactive Decay Law

The radioactive decay law was first established experimentally near the beginning of the century by Rutherford and Soddy and states that the activity of a radioactive sample decays exponentially in time. In terms of modern quantum mechanics, this can easily be derived by considering the fact that a nuclear decay process is governed by a transition probability per unit time,  $\lambda$ , characteristic of the nuclear species. If a nuclide has more than one mode of decay, then  $\lambda$  is the sum of the separate constants for each mode

$$\lambda = \lambda_1 + \lambda_2 + \dots \tag{1.11}$$

In a sample of N such nuclei, the *mean* number of nuclei decaying in a time dt would then be

$$\sigma = dN = -\lambda N dt, \qquad (1.12)$$

where N is the number of nuclei and  $\lambda$  is the decay constant. We have assumed here that N is large so that it may be considered as continuous. Equation (1.12) may be considered as the differential form of the radioactive decay law. Integrating (1.12) then results in the exponential,

$$N(t) = N(0) \exp(-\lambda t), \qquad (1.13)$$

where N(0) is the number of the nuclei at t = 0. The exponential decrease in activity of

a radioactive sample is thus governed by the constant  $\lambda$ . In practice, it is more habitual to use the inverse of  $\lambda$ ,

$$\tau_{\rm m} = 1/\lambda \,, \tag{1.14}$$

which is known as the *mean lifetime*. This is just the time it takes for the sample to decay to 1/e of its initial activity. Equally in use is the *half-life*,  $T_{1/2}$ , which is defined as the time it takes for the sample to decay to one-half of its original activity. Thus,

$$\frac{1}{2} = \exp(-\lambda T_{1/2}), \tag{1.15}$$

which implies

$$T_{1/2} = \frac{1}{\lambda} \ln 2 = \tau_{\rm m} \ln 2$$
 (1.16)

#### 1.11.1 Fluctuations in Radioactive Decay

Consider now the number of decays undergone by a radioactive source in a period of time  $\Delta t$  which is short compared to the half-life of the source. The activity of the source may then be considered as constant. If repeated measurements of the number of decays, n, in the interval  $\Delta t$  are now made, fluctuations will be observed from measurement to measurement. This is due to the statistical nature of the decay process; indeed, from quantum mechanics we know that the exact number of decays at any given time can never be predicted, only the probability of such an event. From the radioactive decay law, it can be shown, in fact, (see Segre [1.5], for example) that the probability of observing n counts in a period  $\Delta t$  is given by a Poisson distribution,

$$P(n, \Delta t) = \frac{m^n}{n!} \exp(-m), \qquad (1.17)$$

where m is the average number of counts in the period  $\Delta t$ . The standard deviation of the distribution is then

$$\sigma = \sqrt{m} \tag{1.18}$$

as is characteristic of Poisson statistics.

**Example 1.1** A source is observed for a period of 5 s during which 900 counts are accumulated by the detector. What is the count rate per second and error from this measurement?

Take the measurement as a single trial for the determination of the mean count rate in 5 s; i.e., m = 900 for  $\Delta t = 5$  s. The standard deviation is then

$$\sigma = \sqrt{900} = 30.$$

The count rate per second is then

rate/s = 
$$(900 \pm 30)/5 = (180 \pm 6)$$
 cts/s.